宿題 6

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For a given $x(0) = x_0 \in \mathbb{R}^n$, let us consider the infinite horizon optimal control problem of discrete-time systems.

$$\inf_{\substack{u(\tau)\in\mathbb{R}^m \\ \tau\in\mathbb{Z}^+}} J(x_0; u(\cdot))$$

where

$$J(x_0; u(\cdot)) = \sum_{\tau=0}^{\infty} \ell(x(\tau), u(\tau))$$

and

$$\begin{aligned} x(t+1) = f(x(t), u(t)) \qquad & x(0) = x_0 \qquad t \in \mathbb{Z}^+ = \{0, 1, 2, \dots\} \\ & x(t) \in \mathbb{R}^n \qquad u(t) \in \mathbb{R}^m \end{aligned}$$

A solution, optimal control input, can be given in a state feedback form

$$u(t) = u(x(t)) = \arg\min_{u\in\mathbb{R}^m} \{\ell(x(t), u) + V(f(x(t), u)\}$$

where $V : \mathbb{R}^n \to \mathbb{R}$ is a solution to the following Bellman equation.

$$V(x) = \inf_{u\in\mathbb{R}^m} \{\ell(x, u) + V(f(x, u))\} \qquad \text{for all } x \in \mathbb{R}^n$$

Let us now consider linear discrete-time systems.

$$x(t+1) = Ax(t) + Bu(t) \qquad x(0) = x_0 \qquad t \in \mathbb{Z}^+ = \{\, 0, 1, 2, \dots \,\}$$
$$x(t) \in \mathbb{R}^n \qquad u(t) \in \mathbb{R}^m$$
$$A \in \mathbb{R}^{n\times n} \qquad B \in \mathbb{R}^{n\times m}$$

For a given $x(0) = x_0 \in \mathbb{R}^n$, define the quadratic cost function

$$J(x_0; u(\cdot)) = \sum_{\tau=0}^{\infty} (x^{\mathrm{T}}(\tau) Q x(\tau) + u^{\mathrm{T}}(\tau) R u(\tau))$$
$$Q \in \mathbb{R}^{n\times n}, Q = Q^{\mathrm{T}} \geq 0 \qquad R \in \mathbb{R}^{m\times m}, R = R^{\mathrm{T}} > 0$$

and consider the optimal control problem

$$\inf_{\substack{u(\tau)\in\mathbb{R}^m \\ \tau\in\mathbb{Z}^+}} J(x_0; u(\cdot))$$

problem: Show that a solution, optimal control input, will be given in a state feedback form

$$u(t) = -(B^{\mathrm{T}} P B + R)^{-1} B^{\mathrm{T}} P A x(t)$$

by using a solution $P \in \mathbb{R}^{n\times n}$, $P = P^{\mathrm{T}} > 0$ to discrete-time Riccati equation

$$P = A^{\mathrm{T}} P A - A^{\mathrm{T}} P B (B^{\mathrm{T}} P B + R)^{-1} B^{\mathrm{T}} P A + Q$$

hint: You can use the fact that the cost-to-go, $V(x)$, for the LQ problem is actually given as a quadratic function of the state x, i.e., $V(x) = x^{\mathrm{T}} P x$.