Advanced Control Systems Engineering I:

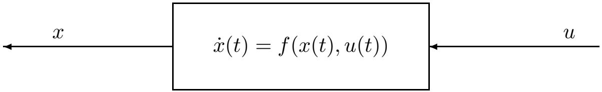
Optimal Control

contents

optimal control

nonlinear dynamical systems and linear approximations

$$\dot{x}(t) = f(x(t), u(t)) \qquad x(t) \in \mathbb{R}^n \quad u(t) \in \mathbb{R}^m$$



nonlinear dynamical systems and linear approximations

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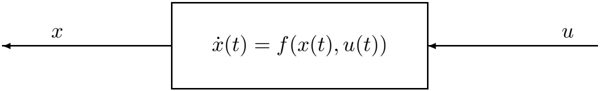
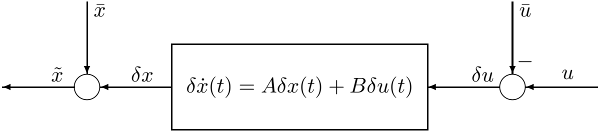


$$\delta\dot{x}(t) = A\delta x(t) + B\delta u(t) \qquad A \in \mathbb{R}^{n\times n} \quad B \in \mathbb{R}^{n\times m}$$



nonlinear dynamical systems and linear approximations

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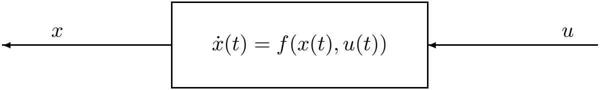

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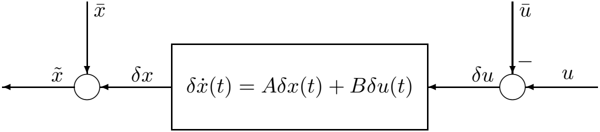


$$A = \left(\frac{\partial f}{\partial x}(\bar{x}, \bar{u})\right)^{\mathrm{T}} \qquad B = \left(\frac{\partial f}{\partial u}(\bar{x}, \bar{u})\right)^{\mathrm{T}}$$

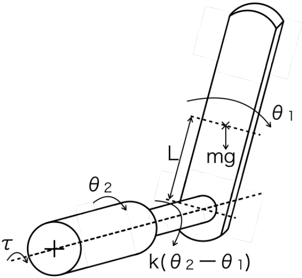
dynamical systems

$$\dot{x}(t) = f(x(t), u(t)) \qquad x(t_0) = x_0 \qquad t \in [\, t_0, t_\mathrm{f} \,]$$

$$x(t) \in \mathbb{R}^n \qquad u(t) \in \mathbb{R}^m$$

robotic manipulator

dynamical systems



$$J_1\ddot{\theta}_1(t) = k(\theta_2(t) - \theta_1(t)) + mgL\sin\theta_1(t)$$
$$J_2\ddot{\theta}_2 = \tau(t) - k(\theta_2(t) - \theta_1(t))$$

state equation

robotic manipulator

$$\begin{aligned} J_1\ddot{\theta}_1(t) &= k(\theta_2(t) - \theta_1(t)) + mgL\sin\theta_1(t) \\ J_2\ddot{\theta}_2 &= \tau(t) - k(\theta_2(t) - \theta_1(t)) \end{aligned}$$

state equation

robotic manipulator

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state equation

robotic manipulator

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$$\dot{x}(t) = f(x(t), u(t)) \qquad x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad u = \tau$$

state equation

robotic manipulator

$$J_1\ddot{\theta}_1(t) = k(\theta_2(t) - \theta_1(t)) + mgL\sin\theta_1(t)$$
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$$\dot{x}(t) = f(x(t), u(t)) \qquad x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad u = \tau$$

$$\frac{d}{dt}\begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1 \\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x, u) \end{bmatrix}$$

equilibrium point

nonlinear dynamical systems and linear approximations

$$\dot{x}(t) = f(x(t), u(t))$$

equilibrium point

nonlinear dynamical systems and linear approximations

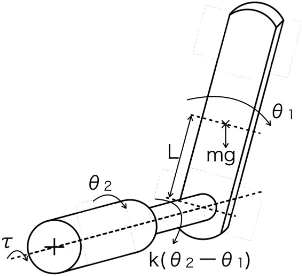
$$\dot{x}(t) = f(x(t), u(t))$$

a pair $(\bar{x}, \bar{u})$ is an equilibrium point if

$$0 = f(\bar{x}, \bar{u})$$

equilibrium point

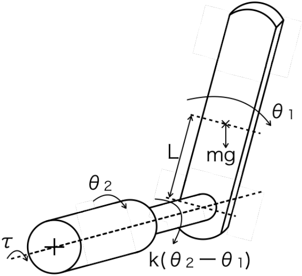
robotic manipulator



$$\frac{d}{dt}\begin{bmatrix}\theta_1\\ \theta_2\\ \dot\theta_1\\ \dot\theta_2\end{bmatrix} = \begin{bmatrix}\dot\theta_1\\ \dot\theta_2\\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1\\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau\end{bmatrix}$$

equilibrium point

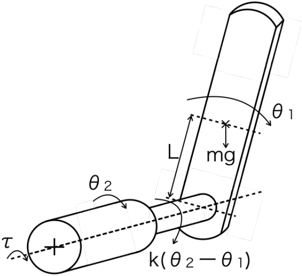
robotic manipulator



$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1 \\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau \end{bmatrix}$$

equilibrium point

robotic manipulator

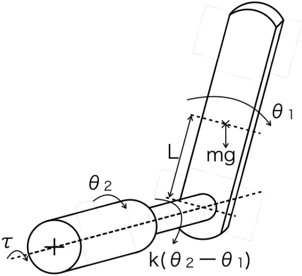


$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1\\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau\end{bmatrix}$$

$$(\bar{x}, \bar{u}) = (\begin{bmatrix}\pi\\ \pi\\ 0\\ 0\end{bmatrix}, 0)$$

equilibrium point

robotic manipulator



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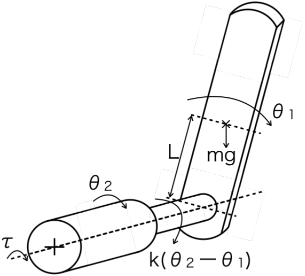
$$(\bar{x},\bar{u}) = (\begin{bmatrix}\pi\\ \pi\\ 0\\ 0\end{bmatrix}, 0) \qquad (\bar{x},\bar{u}) = (\begin{bmatrix}0\\0\\0\\0\end{bmatrix}, 0)$$

equilibrium point

robotic manipulator



$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1\\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau\end{bmatrix}$$

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} q\\ q - \frac{mgL}{k}\sin q\\ 0\\ 0\end{bmatrix}, -mgL\sin q\right) \qquad \text{for all } q \in [\,\pi, -\pi\,)$$

equilibrium point

$$\dot{x}(t) = f(x(t), u(t))$$

a pair $(\bar{x}, \bar{u})$ is an equilibrium point if

$$0 = f(\bar{x}, \bar{u})$$

notations

$$h:\ \mathbb{R} \to \mathbb{R} \qquad y = h(x)$$

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$$\frac{dh}{dx}(x)$$

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$$y + \delta y = h(x + \delta x)$$

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$$h : \mathbb{R} \to \mathbb{R} \qquad y = h(x)$$

$$\frac{dh}{dx}(x)$$

$$\begin{aligned} y + \delta y &= h(x + \delta x) \\ &= h(x) + \frac{dh}{dx}(x)\delta x + o(\delta x) \end{aligned}$$

notations

$$h: \mathbb{R} \to \mathbb{R} \qquad y = h(x)$$

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$$\delta y \approx \frac{dh}{dx}(x)\delta x$$

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$$\frac{\partial h}{\partial x_1}(x) \qquad \frac{\partial h}{\partial x_2}(x) \qquad \cdots \qquad \frac{\partial h}{\partial x_n}(x)$$

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$$\frac{\partial h}{\partial x}(x) =$$

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$$h: \mathbb{R}^n \to \mathbb{R} \qquad y = h(x) \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

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$$\begin{aligned} y + \delta y &= h(x + \delta x) \\ &= h(x) + \frac{\partial h}{\partial x_1}(x)\delta x_1 + \frac{\partial h}{\partial x_2}(x)\delta x_2 + \cdots + \frac{\partial h}{\partial x_n}(x)\delta x_n \\ &\qquad + o(\delta x_1) + o(\delta x_2) + \cdots o(\delta x_n) \\ &= \end{aligned}$$

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$$h : \mathbb{R}^n \to \mathbb{R} \qquad y = h(x) \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \delta x = \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_n \end{bmatrix}$$

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notations

$$h:\ \mathbb{R}^n \to \mathbb{R}^m \qquad y = h(x)$$

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$$h:\ \mathbb{R}^n \to \mathbb{R}^m \qquad y = h(x) \qquad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_m(x) \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

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$$\frac{\partial h_1}{\partial x_1}(x) \qquad \frac{\partial h_1}{\partial x_2}(x) \qquad \cdots \qquad \frac{\partial h_1}{\partial x_n}(x)$$

notations

$$h : \mathbb{R}^n \to \mathbb{R}^m \qquad y = h(x) \qquad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_m(x) \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\begin{matrix} \frac{\partial h_1}{\partial x_1}(x) & \frac{\partial h_1}{\partial x_2}(x) & \cdots & \frac{\partial h_1}{\partial x_n}(x) \\ \frac{\partial h_2}{\partial x_1}(x) & \frac{\partial h_2}{\partial x_2}(x) & \cdots & \frac{\partial h_2}{\partial x_n}(x) \\ \vdots & & & \end{matrix}$$

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$$\frac{\partial h}{\partial x}(x) =$$

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$$h : \mathbb{R}^n \to \mathbb{R}^m \qquad y = h(x) \qquad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_m(x) \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

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$$\frac{\partial h}{\partial x}(x) = \begin{bmatrix} \frac{\partial h_1}{\partial x}(x) & \frac{\partial h_2}{\partial x}(x) & \cdots & \frac{\partial h_m}{\partial x}(x) \end{bmatrix} \in \mathbb{R}^{n \times m}$$

notations

$$h: \mathbb{R}^n \to \mathbb{R}^m \qquad y = h(x)$$

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$$h:\ \mathbb{R}^n \to \mathbb{R}^m \qquad y = h(x)$$

$$\begin{aligned}
\frac{\partial h}{\partial x}(x) &= \begin{bmatrix} \frac{\partial h_1}{\partial x}(x) & \frac{\partial h_2}{\partial x}(x) & \cdots & \frac{\partial h_m}{\partial x}(x) \end{bmatrix} \\
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\end{aligned}$$

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$$h: \mathbb{R}^n \to \mathbb{R}^m \quad y = h(x) \quad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_m(x) \end{bmatrix} \; x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \; \delta x = \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_n \end{bmatrix}$$

$$y + \delta y = h(x + \delta x)$$

notations

$$h : \mathbb{R}^n \to \mathbb{R}^m \quad y = h(x) \quad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_m(x) \end{bmatrix} \; x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \; \delta x = \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_n \end{bmatrix}$$

$$y + \delta y = h(x + \delta x)$$

$$\begin{bmatrix} \delta y_1 \\ \delta y_2 \\ \vdots \\ \delta y_m \end{bmatrix} \approx$$

notations

$$h:\ \mathbb{R}^n \to \mathbb{R}^m \quad y = h(x) \quad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_m(x) \end{bmatrix} \; x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix},\ \delta x = \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_n \end{bmatrix}$$

$$y + \delta y = h(x + \delta x)$$

$$\begin{bmatrix} \delta y_1 \\ \delta y_2 \\ \vdots \\ \delta y_m \end{bmatrix} \approx \begin{bmatrix} \left(\frac{\partial h_1}{\partial x}(x)\right)^{\mathrm{T}} \delta x \\ \left(\frac{\partial h_2}{\partial x}(x)\right)^{\mathrm{T}} \delta x \\ \vdots \\ \left(\frac{\partial h_m}{\partial x}(x)\right)^{\mathrm{T}} \delta x \end{bmatrix} =$$

notations

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$$=$$

notations

$$h: \mathbb{R}^n \to \mathbb{R}^m \quad y = h(x) \quad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_m(x) \end{bmatrix} \; x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \; \delta x = \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_n \end{bmatrix}$$

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$$= \begin{bmatrix} \frac{\partial h_1}{\partial x}(x) & \frac{\partial h_2}{\partial x}(x) & \cdots & \frac{\partial h_m}{\partial x}(x) \end{bmatrix}^{\mathrm{T}} \delta x =$$

notations

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$$= \begin{bmatrix} \frac{\partial h_1}{\partial x}(x) & \frac{\partial h_2}{\partial x}(x) & \cdots & \frac{\partial h_m}{\partial x}(x) \end{bmatrix}^{\mathrm{T}} \delta x = \left(\frac{\partial h}{\partial x}(x)\right)^{\mathrm{T}} \delta x$$

notations

$$h:\ \mathbb{R} \to \mathbb{R} \qquad y = h(y) \quad \delta y \approx \frac{dh}{dx}(x)\delta x$$

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$$h : \mathbb{R} \to \mathbb{R} \qquad y = h(y) \quad \delta y \approx \frac{dh}{dx}(x)\delta x$$

$$h : \mathbb{R}^n \to \mathbb{R} \qquad y = h(y) \quad \delta y \approx \left(\frac{\partial h}{\partial x}(x)\right)^{\mathrm{T}} \delta x \quad \frac{\partial h}{\partial x}(x) \in \mathbb{R}^{n\times 1}$$

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linear approximations

nonlinear dynamical systems and linear approximations

$$\dot{x}(t) = f(x(t), u(t)) \qquad f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$$

linear approximations

nonlinear dynamical systems and linear approximations

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linear approximations

nonlinear dynamical systems and linear approximations

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$$f(x + \delta x, u + \delta u) =$$

linear approximations

nonlinear dynamical systems and linear approximations

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$$f(x, u)$$

$$f(x + \delta x, u + \delta u) = f(x, u) + \left(\frac{\partial f}{\partial x}(x, u)\right)^{\mathrm{T}} \delta x + \left(\frac{\partial f}{\partial u}(x, u)\right)^{\mathrm{T}} \delta u + o(\delta x) + o(\delta u)$$

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linear approximations

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pick an equilibrium point $(\bar{x}, \bar{u})$ $\qquad 0 = f(\bar{x}, \bar{u})$

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small deviation around the equilibrium point

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linear approximations

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linear approximations

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$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ \frac{d}{dt}(\bar{x} + \delta x(t)) &= f(\bar{x} + \delta x(t), \bar{u} + \delta u(t)) \end{aligned}$$

linear approximations

nonlinear dynamical systems and linear approximations

$$\dot{x}(t) = f(x(t), u(t)) \qquad f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$$

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linear approximations

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$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ \frac{d}{dt}(\bar{x} + \delta x(t)) &= f(\bar{x} + \delta x(t), \bar{u} + \delta u(t)) \\ \delta \dot{x}(t) &= \underbrace{f(\bar{x}, \bar{u})}_{=0} + \underbrace{\left(\frac{\partial f}{\partial x}(\bar{x}, \bar{u})\right)^{\mathrm{T}}}_{=A} \delta x(t) + \underbrace{\left(\frac{\partial f}{\partial u}(\bar{x}, \bar{u})\right)^{\mathrm{T}}}_{=B} \delta u(t) \\ &\quad + o(\delta x(t)) + o(\delta u(t)) \end{aligned}$$

linear approximations

nonlinear dynamical systems and linear approximations

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pick an equilibrium point $(\bar{x}, \bar{u})$ $\qquad 0 = f(\bar{x}, \bar{u})$

small deviation around the equilibrium point

$$x(t) = \bar{x} + \delta x(t) \qquad u(t) = \bar{u} + \delta u(t)$$

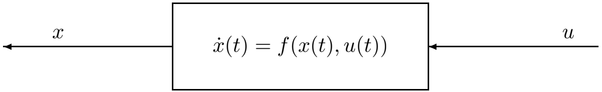
linear approximation around the equilibrium point

$$\delta \dot{x}(t) = A\delta x(t) + B\delta u(t)$$

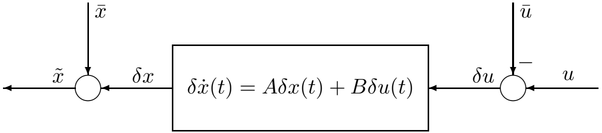
$$A = \left(\frac{\partial f}{\partial x}(\bar{x}, \bar{u})\right)^{\mathrm{T}} \in \mathbb{R}^{n\times n} \qquad B = \left(\frac{\partial f}{\partial u}(\bar{x}, \bar{u})\right)^{\mathrm{T}} \in \mathbb{R}^{n\times m}$$

nonlinear dynamical systems and linear approximations

$$\dot{x}(t) = f(x(t), u(t)) \qquad x(t) \in \mathbb{R}^n \quad u(t) \in \mathbb{R}^m$$



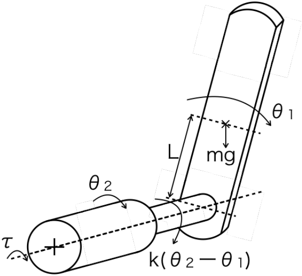
$$\delta\dot{x}(t) = A\delta x(t) + B\delta u(t) \qquad A \in \mathbb{R}^{n\times n} \quad B \in \mathbb{R}^{n\times m}$$



$$A = \left(\frac{\partial f}{\partial x}(\bar{x}, \bar{u})\right)^{\mathrm{T}} \qquad B = \left(\frac{\partial f}{\partial u}(\bar{x}, \bar{u})\right)^{\mathrm{T}}$$

equilibrium point

robotic manipulator



$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1 \\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau \end{bmatrix}$$

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} q \\ q - \frac{mgL}{k}\sin q \\ 0 \\ 0 \end{bmatrix}, -mgL\sin q \right) \qquad \text{for all } q \in [\,\pi, -\pi\,)$$

state equation

robotic manipulator

$$J_1\ddot{\theta}_1(t) = k(\theta_2(t) - \theta_1(t)) + mgL\sin\theta_1(t)$$
$$J_2\ddot{\theta}_2 = \tau(t) - k(\theta_2(t) - \theta_1(t))$$

$$\dot{x}(t) = f(x(t), u(t)) \qquad x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \qquad u = \tau$$

$$\frac{d}{dt}\begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1 \\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x, u) \end{bmatrix}$$

linear approximation

robotic manipulator

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 0 & -\frac{k}{J_1} + \frac{mgL}{J_1}\cos\theta_1 & \frac{k}{J_2} \\ 0 & 0 & \frac{k}{J_1} & -\frac{k}{J_2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial f}{\partial u} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{J_2} \end{bmatrix}$$

linear approximation

robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \bar{q} \\ \bar{q} - \frac{mgL}{k} \sin \bar{q} \\ 0 \\ 0 \end{bmatrix}, -mgL \sin \bar{q} \right) \qquad \text{for some } \bar{q} \in [\, \pi, -\pi \,)$$

linear approximation

robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \bar{q} \\ \bar{q} - \frac{mgL}{k} \sin \bar{q} \\ 0 \\ 0 \end{bmatrix}, -mgL \sin \bar{q} \right) \qquad \text{for some } \bar{q} \in [\, \pi, -\pi \,)$$

$$\delta \dot{x}(t) = A \delta x + B \delta u$$

linear approximation

robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \bar{q} \\ \bar{q} - \frac{mgL}{k} \sin \bar{q} \\ 0 \\ 0 \end{bmatrix}, -mgL \sin \bar{q} \right) \qquad \text{for some } \bar{q} \in [\, \pi, -\pi \,)$$

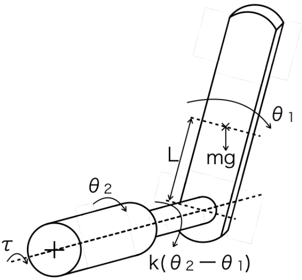
$$\delta \dot{x}(t) = A \delta x + B \delta u$$

$$A = \left(\frac{\partial f}{\partial x}(\bar{x}, \bar{u}) \right)^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{J_1} + \frac{mgL}{J_1} \cos \bar{q} & \frac{k}{J_1} & 0 & 0 \\ \frac{k}{J_2} & -\frac{k}{J_2} & 0 & 0 \end{bmatrix}$$

$$B = \left(\frac{\partial f}{\partial u}(\bar{x}, \bar{u}) \right)^{\mathrm{T}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{bmatrix}$$

numerical simulation

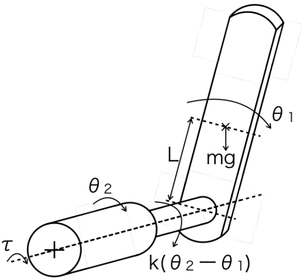
robotic manipulator



$$\frac{d}{dt}\begin{bmatrix}\theta_1\\ \theta_2\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} = \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1\\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau\end{bmatrix}$$

numerical simulation

robotic manipulator

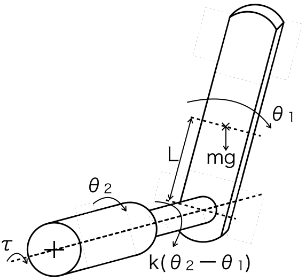


$$\frac{d}{dt}\begin{bmatrix}\theta_1\\ \theta_2\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} = \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1\\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau\end{bmatrix}$$

$$J_1 = J_2 = m = k = L = g = 1$$

numerical simulation

robotic manipulator



$$\frac{d}{dt}\begin{bmatrix}\theta_1\\ \theta_2\\ \dot\theta_1\\ \dot\theta_2\end{bmatrix} = \begin{bmatrix}\dot\theta_1\\ \dot\theta_2\\ -\frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 + \frac{mgL}{J_1}\sin\theta_1\\ \frac{k}{J_2}\theta_1 - \frac{k}{J_2}\theta_2 + \frac{1}{J_2}\tau\end{bmatrix}$$

$$J_1 = J_2 = m = k = L = g = 1$$

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix}\bar{q}\\ \bar{q} - \frac{mgL}{k}\sin\bar{q}\\ 0\\ 0\end{bmatrix}, -mgL\sin\bar{q}\right) \qquad \text{for some } \bar{q} \in [\,\pi, -\pi\,)$$

$$\bar{q} = \pi\ [\,\mathrm{rad}\,]$$

numerical simulation

robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}, 0 \right)$$

numerical simulation

robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}, 0 \right) \quad x(0) = \begin{bmatrix} 11\pi/12 \\ 11\pi/12 \\ 0 \\ 0 \end{bmatrix} \quad \left(\delta x(0) = \begin{bmatrix} -\pi/12 \\ -\pi/12 \\ 0 \\ 0 \end{bmatrix} \right)$$

numerical simulation

robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}, 0 \right) \quad x(0) = \begin{bmatrix} 11\pi/12 \\ 11\pi/12 \\ 0 \\ 0 \end{bmatrix} \quad \left(\delta x(0) = \begin{bmatrix} -\pi/12 \\ -\pi/12 \\ 0 \\ 0 \end{bmatrix} \right) \quad u(\cdot) = 0$$

numerical simulation

robotic manipulator

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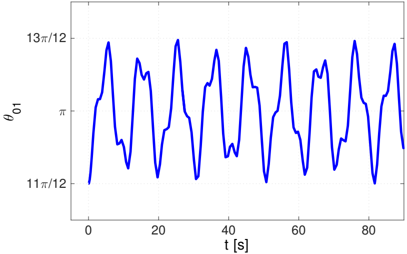
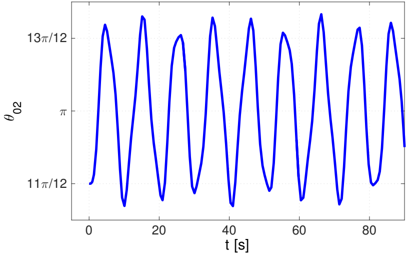
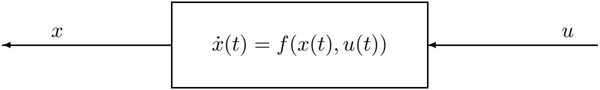


θ_1



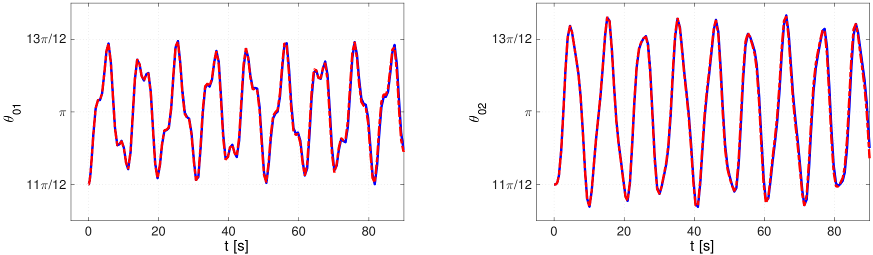
θ_2



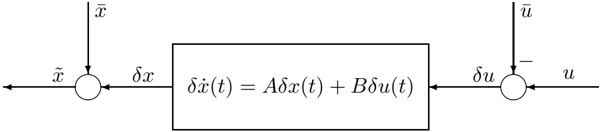
numerical simulation

robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}, 0 \right) \quad x(0) = \begin{bmatrix} 11\pi/12 \\ 11\pi/12 \\ 0 \\ 0 \end{bmatrix} \quad \left(\delta x(0) = \begin{bmatrix} -\pi/12 \\ -\pi/12 \\ 0 \\ 0 \end{bmatrix} \right) \quad u(\cdot) = 0$$



θ_1 θ_2



numerical simulation

robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}, 0 \right)$$

numerical simulation

robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}, 0 \right) \quad x(0) = \begin{bmatrix} \pi/2 \\ \pi/2 \\ 0 \\ 0 \end{bmatrix} \quad \left(\delta x(0) = \begin{bmatrix} -\pi/2 \\ -\pi/2 \\ 0 \\ 0 \end{bmatrix} \right)$$

numerical simulation

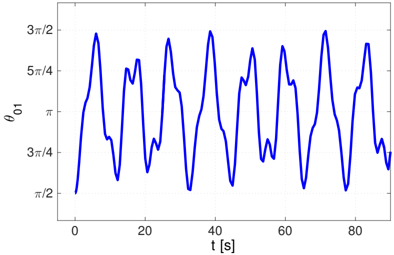
robotic manipulator

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}, 0 \right) \quad x(0) = \begin{bmatrix} \pi/2 \\ \pi/2 \\ 0 \\ 0 \end{bmatrix} \quad \left(\delta x(0) = \begin{bmatrix} -\pi/2 \\ -\pi/2 \\ 0 \\ 0 \end{bmatrix} \right) \quad u(\cdot) = 0$$

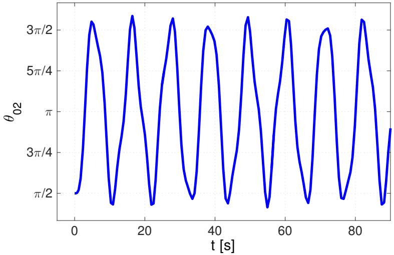
numerical simulation

robotic manipulator

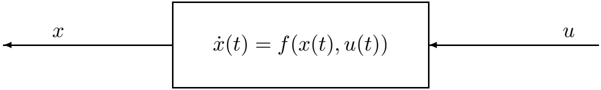
$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}, 0 \right) \quad x(0) = \begin{bmatrix} \pi/2 \\ \pi/2 \\ 0 \\ 0 \end{bmatrix} \quad \left(\delta x(0) = \begin{bmatrix} -\pi/2 \\ -\pi/2 \\ 0 \\ 0 \end{bmatrix} \right) \quad u(\cdot) = 0$$



θ_1



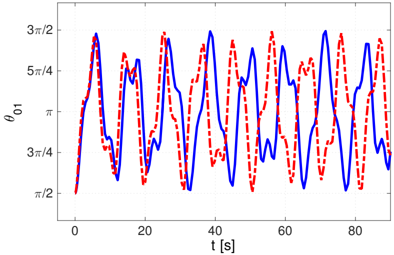
θ_2



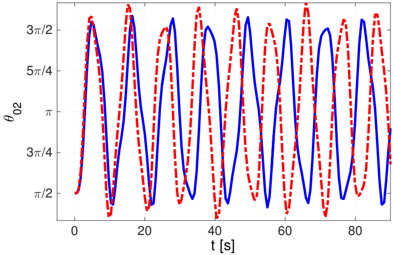
numerical simulation

robotic manipulator

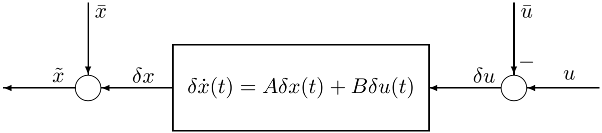
$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}, 0 \right) \quad x(0) = \begin{bmatrix} \pi/2 \\ \pi/2 \\ 0 \\ 0 \end{bmatrix} \quad \left(\delta x(0) = \begin{bmatrix} -\pi/2 \\ -\pi/2 \\ 0 \\ 0 \end{bmatrix} \right) \quad u(\cdot) = 0$$



θ_1

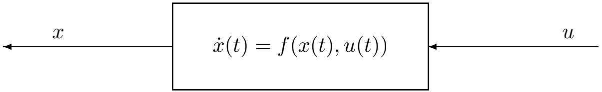


θ_2

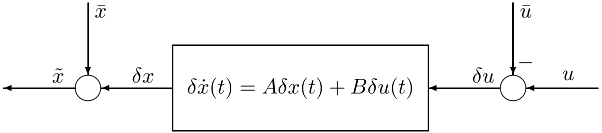


nonlinear dynamical systems and linear approximations

$$\dot{x}(t) = f(x(t), u(t)) \qquad x(t) \in \mathbb{R}^n \quad u(t) \in \mathbb{R}^m$$



$$\delta\dot{x}(t) = A\delta x(t) + B\delta u(t) \qquad A \in \mathbb{R}^{n\times n} \quad B \in \mathbb{R}^{n\times m}$$



$$A = \left(\frac{\partial f}{\partial x}(\bar{x}, \bar{u})\right)^{\mathrm{T}} \qquad B = \left(\frac{\partial f}{\partial u}(\bar{x}, \bar{u})\right)^{\mathrm{T}}$$

contents

optimal control

- nonlinear dynamical systems and linear approximations
- dynamic programming
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- decentralized optimal control
    - decentralization and integration via mechanism design